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Direct continuous phase demodulation in digital holography with use of the transport-of-intensity equation

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ABSTRACT

A novel approach for continuous phase demodulation in digital holography is proposed. This method allows direct recovery of continuous phase information encoded in digital holography. Transport-of-intensity-equation is invoked following the numerical reconstruction and propagation of the digital hologram. The recovered phase is free from the 2π discontinuities and thus phase unwrapping problems are avoided. Furthermore, it provides a new way to eliminate the tilt and quadratic phase aberration inherent in digital holography without cumbersome physical or digital compensation procedure. The performance and feasibility of the method are demonstrated through two applications in micro-optics and bio-imaging.

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1. Introduction

Phase retrieval has been a subject of considerable interest in many fields where either phase imaging or structure retrieval is an issue, such as optical testing, bio-medical imaging and materials science [1–3]. Various interferometric techniques have been developed in the past couple of decades to obtain full field, quantitative, and absolute phase imaging [2,4]. Among these techniques, digital holography has emerged as a front-runner for phase imaging for its attractive ability to reconstruct simultaneously an amplitude-contrast and a phase-contrast image from a single hologram [5,6]. Unlike Fourier phase or Hilbert phase microscopy, digital holography does not require recording in-focus images of the specimen on the digital detector (CCD or CMOS camera), because numerical focusing can be achieved by the numerical wavefront propagation. For an off-axis hologram, the object wave is spatially separated from the zero-order undiffracted wave and the conjugate object wave. Thus, the amplitude and phase information of the object at the hologram plane can be delineated via spatially filtering in the Fourier domain [7]. The filtered hologram is then multiplied by a numerical reference (usually a plane wave) and the resulting wave-field is propagated

to the image plane to determine the resulting in-focus amplitude and phase of the object. However, phase aberrations including tilt due to the off-axis geometry and phase curvature caused by the microscope objective superimposes on the object phase. In order to accurately recover the phase information induced by the object only, a time-consuming and tedious compensation procedure, either physical or numerical, is often performed [8–12]. Another major obstacle that frustrates this and other phase measurement techniques is that the recovered phase is mathematically limited to the interval $(-\pi, \pi]$, corresponding to the principal value of the arctangent function. Therefore phase unwrapping must be carried out before any reconstruction of the physical quantities from the given phase map. Several algorithms for phase unwrapping have been reported in literature to overcome this difficulty [13–15]. However, only a few of them are good enough for practical applications in the presence of noise, rapid phase variations and phase residues. Sometimes not just the unwrapping from the local region is flawed, but phase errors also often propagate outward to rest of the image. Besides, most existing phase unwrapping algorithms are computationally intensive and, thus, difficult to implement in real-time.

Transport of intensity equation (TIE) [16], as a non-interferometric single-beam phase retrieval method, has gained increasing attentions and applications in quantitative phase imaging [17–20]. One of the great advantages of TIE is it needs a minimum of just two intensity measurements of the optical wave at two closely spaced planes perpendicular to the direction of propagation to directly reconstruct

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the continuous phase of the wave, without the need for phase unwrapping [18,21]. Taking advantage of this appealing feature, in this paper, we propose a new fast phase demodulation algorithm for deterministic continuous phase retrieval in digital holography based on solving the TIE from the normalized wavefield obtained during the numerical reconstruction of the hologram. This method allows direct recovery of continuous phase information encoded in digital holography and thus phase unwrapping problems are avoided. Since just one hologram is needed, it facilitates dynamic imaging as well. Furthermore, our method provides a new way to eliminate the tilt and quadratic phase aberration inherent in digital holography without cumbersome physical or numerical compensation procedure. Since the method employs fast Fourier transforms (FFT), it is simple and computationally efficient. Two experiments are carried out to verify the effectiveness of our method.

2. Basic principles of digital holography and transport-of-intensity phase retrieval

In both transmission and reflective setups for digital holography, a coherent laser beam is split into two parts—the reference beam directly illuminates the imaging device. The object beam either passes through or reflects off the sample and interferes with the reference beam at the CCD plane with a small angle to generate an off-axis hologram. The intensity distribution recorded by the camera can simply be written as

$$I_H(x, y) = |O|^2 + |R|^2 + RO^* + R^*O \quad (1)$$

$R(x, y)$ and $O(x, y)$ are the reference and object waves, respectively, $*$ denotes the complex conjugate. This hologram is sampled by the digital camera and then transferred into a computer as an array of numbers. Filtering the hologram's two-dimensional Fourier spectrum can eliminate unwanted zero-order and the twin image term [7,22]. The diffracted field, including amplitude and phase distribution at the image plane is then numerically propagated from the hologram plane using Fresnel transform, convolution, or angular spectrum methods. However, phase aberrations including tilt due to the off-axis geometry and phase curvature caused by the microscope objective superimposes on the object phase, such that the reconstructed phase map can be represented as [5,8,11]

$$\varphi(x, y) = \varphi_o(x, y) + k_x x + k_y y + l_r(x^2 + y^2), \quad (2)$$

where $\varphi_o(x, y)$ is the phase delay introduced by the object. The factors k_x , k_y denote the linear phase difference between the object and reference beam due to the off-axis geometry of the setup. The parameter l_r describes the relative divergence between the object and reference beams due to the mismatch in spherical phase curvature. The phase aberration must be compensated by some physical or numerical method [9–12] in order to accurately recover the phase information induced by the object only. Once the complex field has been calculated at the image plane, the object phase, the arctangent of a ratio of the imaginary part and the real part of the complex amplitude, can be determined. However, the phase mapping is ambiguous for objects of optical depths greater than the wavelength, as absolute phase is wrapped in the intervals of 2π because the arctangent function returns the value of the angle in modulo 2π instead of its absolute value. Therefore, the phase unwrapping procedure has to be applied in order to recover a continuous phase distribution that extends over a range of greater than 2π .

TIE uses only object field intensities at multiple axially displaced planes without any interference with a separate reference beam. The experimental configuration for TIE typically involves a 4f imaging system. By translating the camera or the object, multiple intensity images at different image distance can be

obtained. TIE determines the object-plane phase from the first derivative of intensity in the near Fresnel region [16]

$$-k \frac{\partial I(\mathbf{r})}{\partial z} = \nabla \cdot [I(\mathbf{r}) \nabla \varphi(\mathbf{r})], \quad (3)$$

where I is the in-focus intensity, k is the wave number $= 2\pi/\lambda$, \mathbf{r} is the position vector with spatial coordinates (x, y) , ∇ is the gradient operator with respect to \mathbf{r} . z denotes the wave propagation direction and is perpendicular to the x - y plane. Eq. (3) indicates that the transverse phase of the wave can be deduced from the derivative of intensity along propagation direction. Using this equation the phase distribution can be uniquely recovered without phase unwrapping even if the phase varies over many multiples of 2π [18,21].

3. Direct continuous phase demodulation in digital holography with TIE

It is to be noted that for TIE phase retrieval, the continuous phase can be uniquely determined by solving the partial-differential equation (Eq. (3)), with only the intensity distribution and its derivative with respect to z . Though the intensity derivative cannot be measured directly, it can be estimated by finite difference taken between two closely separated images. Without considering effects of noise, a smaller separation between the two planes yields a more accurate approximation of the phase derivative [19,20]. Conventionally, to acquire the two images with slight defocus, either the camera or the object has to be mechanically translated, which inevitably complicates the data acquisition process and slows down the acquisition speed. Numerical focusing is a unique capability of digital holography, where a single hologram is used to calculate the optical field at any number of image planes, emulating the focusing control of a conventional imaging system. This inspires us to properly combine these two techniques so that merits of both methods can be accrued.

The most direct idea is to numerically propagate the recovered field by digital holography to two distances at and very close to the image plane resulting in the two intensity images with slightly defocus. These two images are used to approximate intensity derivative and used in the TIE algorithm to retrieve phase. The angular-spectrum method [7,11,23] is a preferred approach for this purpose as it maintains the pixel size, has no minimum distance requirement for the reconstruction plane, and is flexible and effective in filtering in Fourier domain. The reconstructed wavefield $U_z(x, y)$ at distance z from the wavefield at the hologram plane $U_0(x, y)$ can be written as [7,11,23]

$$U_z(x, y) = \mathcal{F}^{-1} \{ \mathcal{F} \{ U_0(x, y) \} H(f_x, f_y) \}, \quad (4)$$

where $H(f_x, f_y)$ is the angular spectrum optical transfer function in the spatial frequency domain which is expressed as

$$H(f_x, f_y) = \exp \left[j \frac{2\pi z}{\lambda} \sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2} \right], \quad (5)$$

where f_x and f_y are the spatial frequencies in the x and y directions. \mathcal{F} and \mathcal{F}^{-1} are the Fourier transform and the inverse Fourier transform, respectively. Hence it is possible to numerically reconstruct intensity images at the focused plane and defocused plane with a small defocus distance Δz . These reconstructed intensity images can then be used in the TIE formulation (Eq. (3)) to determine the absolute phase free from 2π discontinuity.

Though this method is based on a sound theory and seems quite straightforward, it is not a good choice for practical implementation. TIE is elliptic partial differential equation for the phase function φ and in order to solve the inverse Laplacian by fast numerical methods (such as FFT) an auxiliary function ψ , which

satisfy $\nabla\psi = I\nabla\varphi$ has to be used to convert TIE to the following two Poisson equations

$$-k\frac{\partial I}{\partial z} = \nabla^2\psi, \tag{6}$$

and

$$\nabla \cdot (I^{-1}\nabla\psi) = \nabla^2\varphi, \tag{7}$$

solving the above two equations involve 8 FFT operations, which is a non-trivial computational load. Besides, the auxiliary function $\nabla\psi = I\nabla\varphi$ inherently induce some error since the rotational term is ignored in the Helmholtz decomposition [24]. In addition, TIE works under the assumption that there are no zeros in the intensity image and proper boundary conditions are needed to obtain a unique solution [25,26]. The intensity images reconstructed by digital holography are usually accompanied with varying degrees of speckle noise, making it difficult to get an exact solution from TIE.

The above difficulties can be overcome by making some modification in the implementation, which can be summarized as the following four steps:

- (1) Calculate the complex wavefield at the focus plane by numerically propagating the filtered hologram as in traditional digital holographic reconstruction.
- (2) Normalize the reconstructed complex field by its amplitude. This step makes the object into a pure-phase object with unit intensity at the focus plane ($I(\mathbf{r}) = 1$, for all pixels).
- (3) Obtain the defocused image $I_{\Delta z}$ by propagating the normalized complex field over a small distance Δz . Note Δz should be limited to a very small value (e.g. 1 μm) to minimize the error caused by neglecting high-order terms in finite-difference approximation of the intensity derivative. Since all intensity images are numerically reconstructed, using a small difference interval does not introduce noise as in real measurement. The multiple-plane methods [19,20] are also applicable to reduce the nonlinearity error in finite-difference, but not recommended for their extra computational cost.
- (4) Finally, the two intensity images serve as the input data to solve the TIE. Since in this case the in-focus image has uniform unit intensity, the TIE (Eq. (3)) can be simplified to a standard Poisson equation [17,19,20]

$$-k\frac{\partial I(\mathbf{r})}{\partial z} = \nabla^2\varphi(\mathbf{r}), \tag{8}$$

where

$$\frac{\partial I(\mathbf{r})}{\partial z} \approx \frac{I_{\Delta z}(\mathbf{r}) - I(\mathbf{r})}{\Delta z}. \tag{9}$$

Eq. (8) can be solved using the FFT based Poisson solver of the form [17]

$$\varphi(\mathbf{r}) = \mathcal{F}^{-1} \left\{ \frac{k\mathcal{F}\{\partial I(\mathbf{r})/\partial z\}}{4\pi^2(f_x^2 + f_y^2)} \right\}. \tag{10}$$

Note the solving the Poisson equation only needs two FFTs, which provides a very fast numerical implementation with very little memory demands. Besides, no auxiliary function is needed and the zero intensity problems can be simultaneously solved by assuming a uniform intensity at the image plane.

Unlike the first method discussed above, we discard the intensity information (assume it is uniform) at the image plane and the second intensity is calculated from the phase distribution only. Apparently, the second intensity distribution obtained is not exactly the same as the real intensity at distance Δz . However, this does not affect the accurate phase retrieval because the test object is deliberately assumed to be a pure phase one with uniform

intensity distribution. Imagine a different object which has the same phase distribution as the actual one under test but with different but uniform intensity distribution, TIE can also extracted the correct phase from the numerical propagated intensity, which is somewhat like performing a numerical simulation on the TIE algorithm. Besides, the angular spectrum method implies spatial periodicity of the complex field which is completely correspondent with the periodic boundary condition inherent in the FFT based Poisson solver [26], therefore the exact solution of TIE can be obtained, and an accurate estimate of the continuous phase can be recovered.

A close inspection of Eq. (8) reveals that the tilt aberration can be automatically removed by our method because a linear phase resulting from the periodic carrier frequency does not introduce any transverse intensity gradient. This is a unique advantage provided by our method, making the subsequent quadratic aberration compensation simpler because only a single parameter l_r in Eq. (2) needs to be adjusted. Furthermore, if the curvature induced by the microscope objective is largely compensated by introducing a same microscope objective or adjustable lens in the reference arm (i.e. only reduced aberrations are introduced), the spherical phase aberration will typically reside in lower spatial frequencies as opposed to the details of the sample which occupy the higher spatial frequencies. In this case, a Tikhonov-regularization treatment [27,28] which is commonly used to remove very low frequency artifacts in TIE can also be adopted to simply suppress the quadratic aberration without additional processing

$$\varphi(\mathbf{r}) = \mathcal{F}^{-1} \left\{ \frac{k(f_x^2 + f_y^2)\mathcal{F}\{\partial I(\mathbf{r})/\partial z\}}{4\pi^2[(f_x^2 + f_y^2)^2 + \gamma]} \right\}, \tag{11}$$

where γ is the regularization parameter (typical value $\gamma = (\max(D_x, D_y))^{-1}$, where D_x and D_y are the physical dimensions of the image) that acts to filter out the slowly varying feature corresponding to the spherical phase aberration.

4. Experiments

Experiments were carried out to verify the feasibility and the effectiveness of the proposed algorithm. As a first example, a regular array of micro-bumps fabricated on a Si substrate was used to demonstrate the power of our approach in the presence of rapid phase variations, vortices and noise. The regularly-patterned surface bumps were formed by use of the laser-induced thermal oxidation under pure O_2 with 0.75 bar pressures. Precise topographic reconstruction of such microstructures is of great interest in different application fields such as integrated optics, microelectronics, and solar cells. Usually the depth profile of such structures is measured by a confocal microscope (see Fig. 1(a) and (b) for the confocal topographic image and a cross-section profile) or atomic force microscope, both of which require time consuming scanning operations. In this work, a common-path digital holography microscope (DHM) based on a single cube beam splitter (SCBS) interferometer [12] was adopted as a characterization tool for the inspection of the structure topography by a relatively fast and non-invasive procedure with only a single acquisition. Since the object beam and reference beam share the same optical path, the wavefront curvatures are physically compensated. In this case, the off-axis tilt could be numerical compensated by spectrum centering [8,9,11] in the numerical reconstruction process effectively. Fig. 1(c) shows the wrapped phase obtained by the traditional holographic reconstruction method. It can be seen that the wrapped phase maps contain abundant phase vortices (features in which a closed-loop excursion around a point produces a 2π difference in phase), causing difficulties in conventional

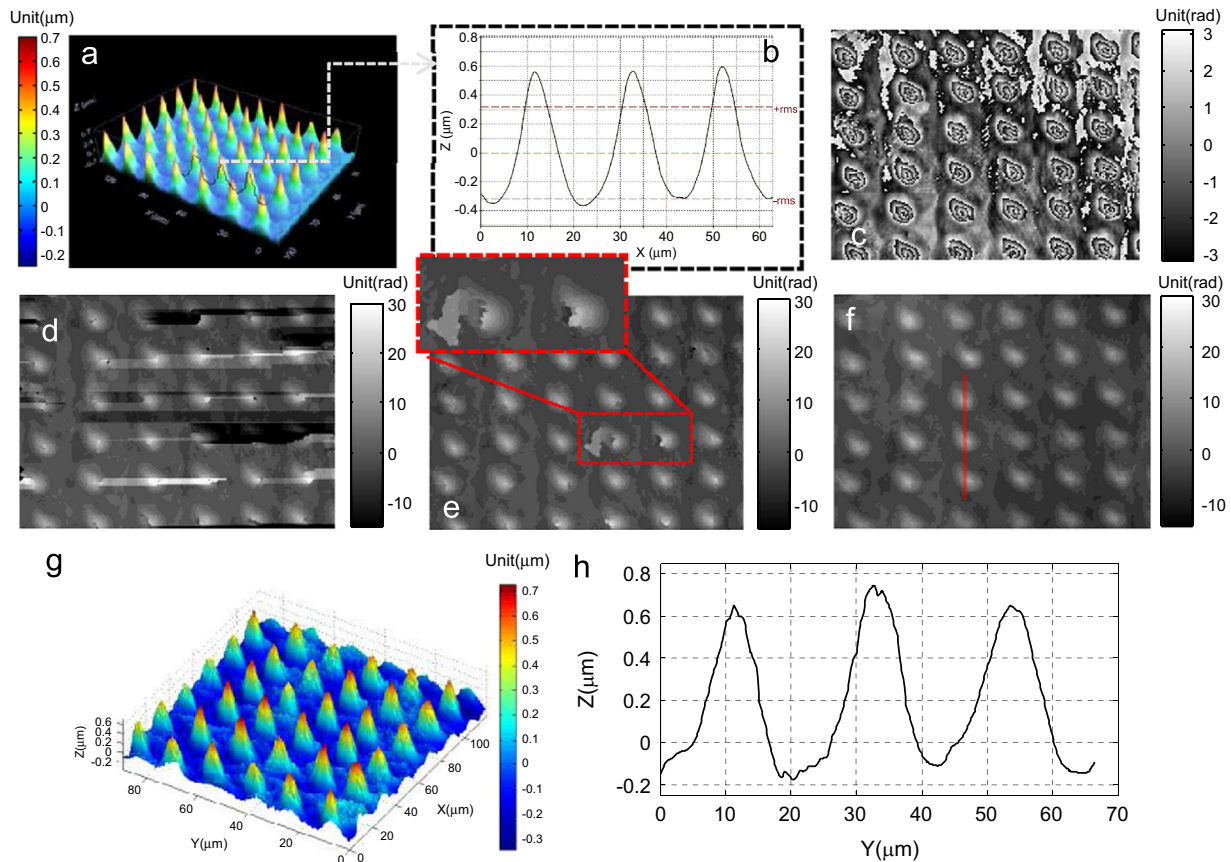


Fig. 1. Experimental results of a regular array of micro-bumps: (a) confocal topographic image; (b) one line profile through the centers of three micro-bumps indicated in (a); (c) wrapped phase image obtained by the SCBS DHM; (d) unwrapped phase obtained by a path-dependent unwrapping method; (e) unwrapped phase obtained by a quality-map guided unwrapping method; (f) continuous phase obtained by our method; (g) 3D visualization of the surface profile; (h) one cross-section of the height distribution along the red line marked in (f).

phase-unwrapping algorithms. Fig. 1(d) shows the unwrapped result obtained by a simple path-dependent phase unwrapping method. Severe line-structure artifacts can be clearly seen caused by error accumulation and propagation. A more complicated quality-guided unwrapping algorithm is capable of preventing these errors from propagating through the image and corrupting good data, but it could not prevent the errors from occurring in the first place (Fig. 1(e)). Some unwrapping artifacts are visibly present in the unwrapped phase map especially around the top regions of the micro-bumps due to rapid phase variations (see the enlarged image). However, using the proposed phase demodulation method, an intact continuous phase map is obtained (Fig. 1(f)), free from any erroneous regions and unwrapping problems. The three-dimensional (3D) topographic image of Fig. 1(f) is shown in Fig. 1(g), by converting the phase map to the physical height of the object. Fig. 1(h) shows the height distribution along one specific line (identified by the red line in Fig. 1(f)), which appears to be consistent with the confocal result. Since our approach requires only four FFTs (two for numerical propagation and another two for solving TIE) with a computational complexity of $4N^2 \log N$, which results in fast processing speeds. The computational time in this example was 321.3 ms for an hologram size of 1280×960 pixels using a 2.5 GHz laptop and in the MATLAB environment. While the conventional quality guided unwrapping method took approximately six times (1.971 s) as long to unwrap the same experimental data. Additionally, since FFT is well suited for parallel computing devices such as graphic processing units (GPUs) [29], it is anticipated that real-time phase reconstruction can be easily realized if GPUs are utilized to solve the TIE algorithm.

A second experiment on a human macrophage cell was performed using a DHM based on a Michelson interferometer geometry [11]. Due to the mismatch between the object and reference curvature, quadratic phase aberration exists, which makes spectrum centering (and hence tilt compensation) less accurate since the spectrum has a broader spread [8,9,11]. The reconstructed wrapped phase after tilt correction (Fig. 2(a)) appear as off-centered concentric circular patterns due to the quadratic phase factor and improper spectrum centering. Fig. 2(b) shows the pseudo-color-encoded unwrapped phase map using the quality guided unwrapping algorithm. The unwrapped phase, aside from some phase unwrapping artifacts, shows a curved background which would affect accurate measurement of sample. To show the ability of the proposed approach to deal with the phase aberration, the proposed algorithm was applied to the filtered hologram directly without any tilt compensation procedure (no spectrum centering). In this case, the unwrapped phase obtained by the traditional method contained a carrier phase with very high frequency, as shown in Fig. 2(c). Since the continuous phase directly recovered by our method assumes a much larger dynamic range, the continuous phase map was rewrapped (see Fig. 2(d)) to permit a clearer illustration and a direct comparison with the wrapped phase shown in Fig. 2(a). It is surprising that the carrier frequency was removed by our method automatically, re-centering the concentric circular patterns to the center of field-of-view. Furthermore, when the Tikhonov-regularization was applied ($\gamma = 2 \times 10^{-3} \mu\text{m}^{-1}$), the curved background could be effectively flattened, as plotted in Fig. 2(e) and (f). Close inspection shows our method extracted the pure phase delay induced by the test object

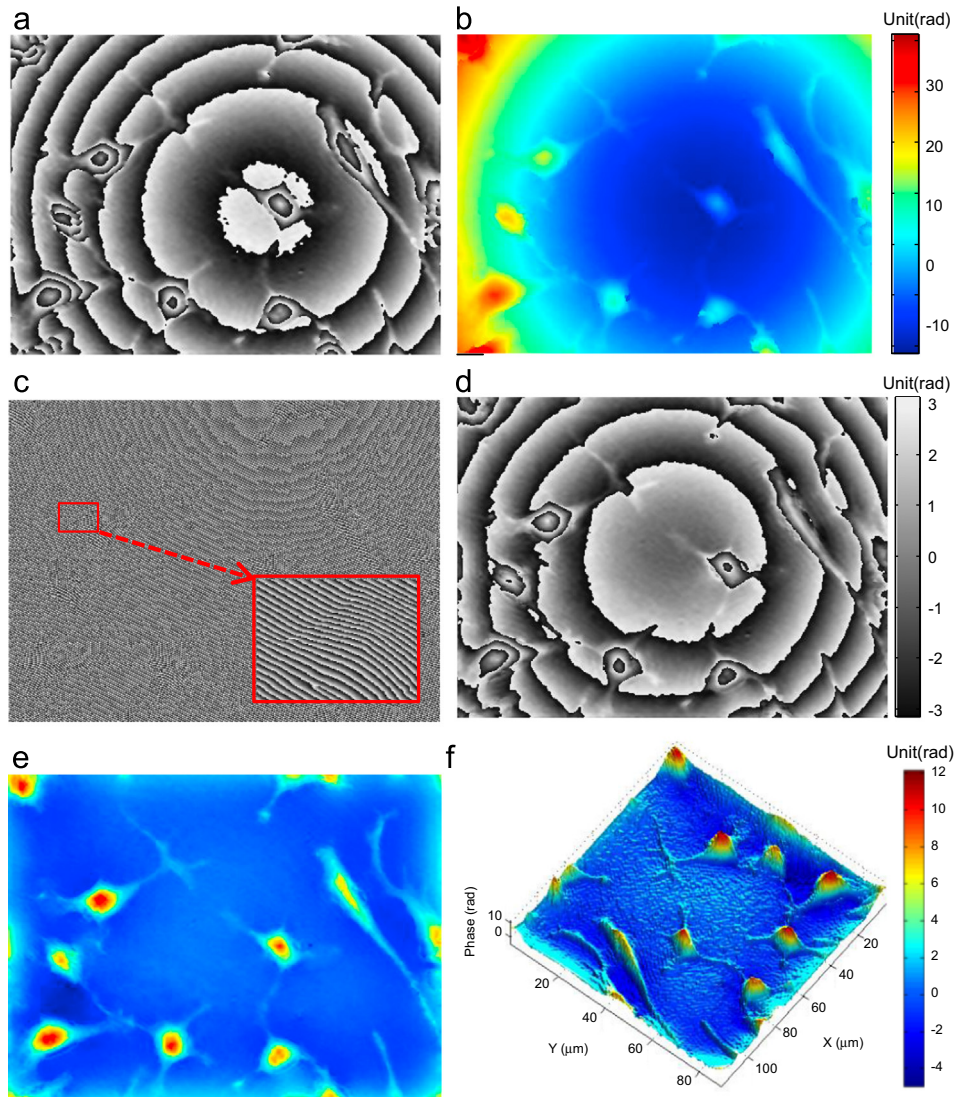


Fig. 2. Experimental results on human macrophage cells; (a) wrapped phase reconstructed after spectrum centering; (b) unwrapped phase map of (a); (c) wrapped phase reconstructed without any tilt compensation; (d) rewrapped phase obtained by the proposed method; (e) continuous phase map obtained by the proposed method with Tikhonov-regularization. (f) 3D rendering of (e).

successfully without reconstruction artifacts such as phase unwrapping errors.

5. Conclusions

In conclusion, a new fast phase demodulation algorithm for deterministic continuous phase retrieval in digital holography based on TIE is proposed. One major advantage of the proposed method is that the absolute phase without 2π discontinuities can be directly recovered. It is simple, reliable, and easy to implement. This provides the opportunity for research to focus on more important issues than the cumbersome task of unwrapping the phase data. Besides, its unique ability to remove the inherent phase aberration in digital holography should be potentially useful for saving cumbersome physical or digital adjusting procedure, increasing the performance of automatic testing, and aiding in visualizing and quantitative analyzing the object phase.

Finally, it should be mentioned that our method can also be applied as a phase unwrapping method which is applicable to many other field such as speckle interferometry, adaptive or compensated optics, magnetic resonance imaging, and synthetic aperture radar

interferometry. Actually, solving the Poisson equation (Eq. (8)) is equivalent to finding a target continuous phase function whose Laplacian approaches the intensity derivative the best (left hand side of Eq. (8)). Therefore, if one wants to use our method as a pure phase unwrapping algorithm, it is recommended that (1) Zero-pad the source image properly in order to avoid artifacts that stem from the periodic nature of the FFT and thus makes the Poisson equation satisfy both Dielectric and Neumann boundary conditions (the phase value and phase gradient are assumed both zeros at the image boundary); (2) The TIE-retrieved phase is only used as a reference phase for determining the number of phase jumps, and the final result is obtained by adding corresponding multiples of 2π to the original unwrapped phase maps.

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