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Range Limited Bi-Histogram Equalization for image contrast enhancement

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ABSTRACT

Histogram equalization is a popular technique for enhancing image contrast. However, it tends to change the brightness of an image and hence, this technique is not very well suited to be implemented in consumer electronics, where preserving the original brightness is essential to avoid annoying artifacts. This paper proposes a novel extension of bi-histogram equalization referred to as Range Limited Bi-Histogram Equalization (RLBHE). First, RLBHE divides the input histogram into two independent sub-histograms by a threshold that minimizes the intra-class variance. This is done in order to effectively separate the objects from the background. Then, range of the equalized image is calculated to yield minimum absolute mean brightness error between the original image and the equalized one. The experimental results show that the proposed method has better performance than the existing methods, and preserve the original brightness quite well, so that it is possible to be utilized in consumer electronic products.

1. Introduction

Global histogram equalization (GHE) is one of the most commonly used methods for image contrast enhancement because it has high efficiency and simplicity. It is achieved by normalizing the intensity distribution using its cumulative distribution function so that the result image may have a uniform distribution of intensity [1].

It is known, however, since GHE is basically using the intensity distribution of the whole image, it may suffers from the some drawbacks such as over enhancement, increase in the noise level, lost in details, and washed-out effect in some almost homogeneous area [2]. So in consumer electronics such as TV, GHE is rarely employed because it may significantly change the brightness of an input image and cause undesirable artifacts.

In the recent years, many researchers proposed many useful algorithms to solve these problems involved in GHE technique. These methods includes Brightness preserving Bi-Histogram Equalization (BBHE) [3], Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE) [4], and Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) [5], etc. BBHE divides the input image histogram into two parts based on the mean of the input image and then each part is equalized independently. It has been analyzed both mathematically and experimentally that this technique is capable to preserve the original brightness to a certain extents. The DSIHE method is similar to BBHE except that it separates the histogram based on the median value. MMBEBHE is another extension of BBHE that provides maximal brightness preservation by using the threshold level, which would yield minimum difference between input and output mean. Though these methods can perform good contrast enhancement, they also cause more annoying side effects depending on the variation of gray level distribution in the histogram. Also RMSHE (Recursive Mean-Separate Histogram Equalization) [6] and RSIHE (Recursive Sub-Image Histogram Equalization) [7] are recursive algorithms of BBHE and DSIHE. These two recursive methods have improved results comparing with previous methods. The mean brightness of the output was similar to that of the input in RMSHE and RSIHE, but the equalization effect was reduced.

This paper presents a new bi-histogram equalization algorithm called Range Limited Bi-Histogram Equalization (RLBHE). This method takes both contrast improvement and brightness preservation into account. To achieve better contrast enhancement and avoid over enhancement, Otsu's method is used to perform histogram thresholding. Then we limit the range of the equalized image to guarantee that the mean output brightness can be almost equal to the mean input brightness. In what follows, GHE and bi-histogram equalization for digital input image is reviewed together with their mathematical formulation in Sections 2 and 3, respectively. The RLBHE method is presented in Section 4. Section 5 lists a few experimental results to illustrate the performance of RLBHE. Section 6 serves as the conclusion of this paper.

2. Global histogram equalization

Let us suppose that $\boldsymbol{X} = \{X(i,j)\}$ denotes a digital image, where $X(i,j)$ denotes the gray level of the pixel at (i,j) place. The total number of the image pixels is n, and the image intensity is digitized

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into L levels that are $\{X_0, X_1, X_2, \ldots, X_{L-1}\}$. So it is obvious that $\forall X(i, j) \in \{X_0, X_1, X_2, \ldots, X_{L-1}\}$. Suppose n_k denotes the total number of pixels with gray level of X_k in the image, then the probability density of X_k will be

$$p(X_k) = \frac{n_k}{n}, \quad k = 0, 1, \ldots, L-1 \tag{1}$$

The relationship between $p(X_k)$ and X_k is defined as the probability density function (PDF), and the graphical appearance of PDF is known as the histogram. Based on the image's PDF, its cumulative distribution function is defined as

$$c(X_k) = \sum_{j=0}^{L-1} p(X_j) = \sum_{j=0}^{L-1} \frac{n_k}{n} \tag{2}$$

where $k = 0, 1, \ldots, L-1$, and it is obvious that $c(X_{L-1}) = 1$. Let us define a transform function $f(x)$ based on the cumulative density function as

$$f(x) = X_0 + (X_{L-1} - X_0)c(x) \tag{3}$$

Then the output image of the GHE, $\boldsymbol{Y} = \{Y(i,j)\}$, can be expressed as

$$\boldsymbol{Y} = f(X) = \{f(X(i,j)|\forall X(i,j) \in \boldsymbol{X}\} \tag{4}$$

It is not difficult to find out that the PDF of the output gray level $\boldsymbol{Y}$ follows a uniform distribution, i.e., the output image should have a density function equally distributed over the entire range, it get the maximum entropy. Suppose that $\boldsymbol{X}$ is a continuous random variable, i.e., $\boldsymbol{L} = \infty$, then the output of the GHE, $\boldsymbol{Y}$ is also regarded as a continuous random variable and

$$p(y) = \frac{1}{X_{L-1} - X_0} \tag{5}$$

Thus, it is easy to show that the mean brightness of the output image of the histogram equalization is the middle gray level since

$$E(\boldsymbol{Y}) = \int_{X_0}^{X_{L-1}} yp(y)dy = \int_{X_0}^{X_{L-1}} \frac{y}{X_{L-1} - X_0} dy = \frac{X_0 + X_{L-1}}{2} \tag{6}$$

where $E(\cdot)$ denotes a statistical expectation. It should be emphasized here that the output mean of the histogram equalization does not take the mean brightness of the original image into account. That is, it is always the middle gray level no matter how much the input image is bright or dark. This property is not desirable in many applications such as consumer electronics because it may significantly change the brightness of an input image and cause undesirable artifacts.

3. Bi-histogram equalization

Many bi-histogram equalization methods have been proposed to overcome the aforementioned problems. Fundamentally, these methods separate the input histogram into two subsections. These two parts are then equalized independently. The major difference among the methods in this family is the criteria used to chose the threshold for separation denoted by X_T. Obviously, $X_T \in \{X_0, X_1, \ldots, X_{L-1}\}$. based on the threshold, the input image $\boldsymbol{X}$ can be decomposed into two sub-images $\boldsymbol{X_L}$ and $\boldsymbol{X_U}$ as

$$X = \boldsymbol{X_L} \cup \boldsymbol{X_U} \tag{7}$$

where

$$\boldsymbol{X_L} = \{X(i,j)|X(i,j) \leq X_T, \forall X(i,j) \in \boldsymbol{X}\} \tag{8}$$

and

$$\boldsymbol{X_U} = \{X(i,j)|X(i,j) > X_T, \forall X(i,j) \in \boldsymbol{X}\} \tag{9}$$

Next, define the respective PDF of the sub-images $\boldsymbol{X_L}$ and $\boldsymbol{X_U}$ as

$$p_L(X_k) = \frac{n_k}{n_L}, \quad k = 0, 1, \ldots, T \tag{10}$$

and

$$p_U(X_k) = \frac{n_k}{n_U}, \quad k = T+1, 1, \ldots, L-1 \tag{11}$$

n_k represent the numbers of X_k in $\boldsymbol{X_L}$ and $\boldsymbol{X_U}$, and n_L and n_U are the total number of samples in $\boldsymbol{X_L}$ and $\boldsymbol{X_U}$, respectively. The respective cumulative density functions for $\boldsymbol{X_L}$ and $\boldsymbol{X_U}$ are then defined as

$$c_L(X_k) = \sum_{j=0}^{k} p_L(X_j) \tag{12}$$

and

$$c_U(X_k) = \sum_{j=T+1}^{k} p_U(X_j) \tag{13}$$

Similar to the case of GHE where a cumulative density function is used as a transform function, let us define the following transform functions exploiting the cumulative density functions

$$f_L(X_k) = X_0 + (X_T - X_0)c_L(X_k), \quad k = 0, 1, \ldots, T \tag{14}$$

and

$$f_U(X_k) = X_{T+1} + (X_{L-1} - X_{T+1})c_U(X_k), \quad k = T+1, 1, \ldots, L-1 \tag{15}$$

Turning the attention to BBHE [3], BBHE separates the input histogram into two parts based on the threshold which is the mean brightness of the input

$$X_T = X_m = \sum_{j=0}^{L-1} X_j p(X_j) \tag{16}$$

Suppose that the input histogram has a symmetrical distribution around its mean. When the sub-images are equalized independently, the mean brightness of the output of the BBHE can be expressed as [3]

$$E(\boldsymbol{Y}) = \frac{X_m}{2} + \frac{X_0 + X_{L-1}}{4} \tag{17}$$

We can see that BBHE can preserve the mean brightness to some extent. But we cannot guarantee every image has the property that the histogram has a quasi-symmetrical distribution around its mean, so its mean brightness preserving was depended on input image. DSIHE [4] is very similar to BBHE, except that the separating point X_D is selected as the median gray level of the input image, i.e., X_D satisfies

$$\sum_{j=0}^{D} p(X_j) \approx \frac{1}{2} \tag{18}$$

It can be proved that the mean brightness of the output image follows

$$E(\boldsymbol{Y}) = \frac{X_D}{2} + \frac{X_0 + X_{L-1}}{4} \tag{19}$$

MMBEBHE [5] is to perform the separation based on the threshold level, which would yield minimum difference between input and output mean. This threshold level is essentially chosen by enumeration. As can be seen from the above discussion, these bi-histogram equalization methods are all determined by the separation threshold. If the threshold is selected, the rest of the procedures are the same. Obviously, we want to get good contrast enhancement results and maintain the brightness of the original

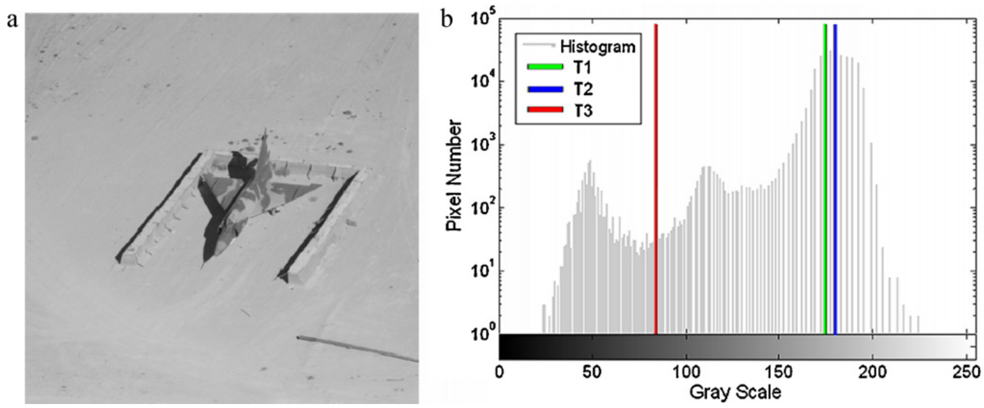

Fig. 1. (a) Original image of *Aircraft*. (b) The locations of thresholds using BBHE (T1), DSIHE (T2), and MMBEBHE (T3) in the histogram of (a).

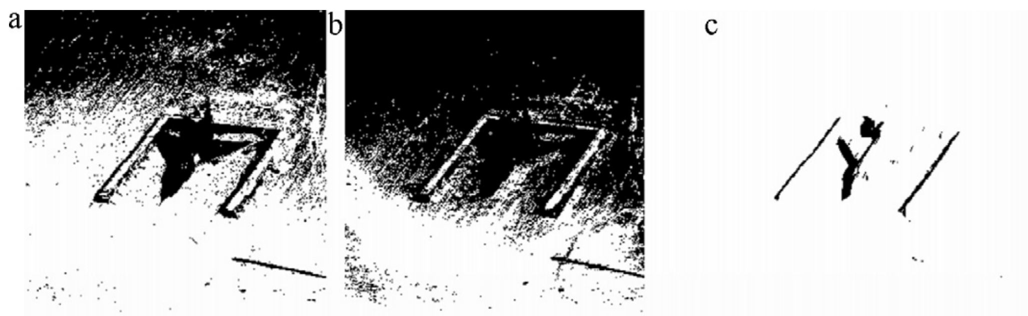

Fig. 2. Image separation results. (a) Separation result of BBHE. (b) Separation result of DSIHE. (c) Separation result of MMBBHE.

image as well. It is suspected that whether there is always an appropriate threshold which could address both aspects.

To demonstrate by an example, Fig. 1 gives a test image where an airplane is located on a very simple background. The object of interest occupies only a small portion of the image. The histogram of Fig. 1(a) (half-logarithm coordinate) and the separation thresholds using BBHE, DISHE, and MMBEBHE are shown in Fig. 2(b). It can be seen from the histogram that the gray scales are mostly concentrated on the range between 150 and 200. This range is corresponding to the background area. We know from Eqs. (2) and (3) that histogram equalization relate the degree of enhancement for a specific range of gray levels with their area (occurrence times). Therefore, the more frequent the gray values occur in a image, the more they will be enhanced, whereas the gray levels with smaller area will be compressed, or even be merged together. So if we apply GHE to this test image, the contrast of the background will be over enhanced and the contrast of the airplane will be suppressed a lot. At this point, the most effective solution is to divide the histogram into two parts using a proper threshold which can separate the background and target effectively. Then use bi-histogram equalization to enhance the two parts independently. Fig. 2 gives the separation results using the thresholds shown in Fig. 1(b). It is clearly that the thresholds of BBHE, DSIHE, and MMBEBHE fail to separate the airplane from its background.

4. Range Limited Bi-Histogram Equalization

RLBHE is formally defined by the following procedures:

1. Choosing a proper threshold for histogram separation
2. Determine the upper and the lower bounds for histogram equalization
3. Equalize each partition independently.

The details of each step are described in the following subsections.

4.1. Choosing a proper threshold for histogram separation

If a threshold used to divide the histogram of Fig. 1(a) into two parts, of course, the most appropriate threshold should be between the lower gray level of the airplane and the higher gray level of the background. Then the target region and the background can be equalized separately, so that the contrast of target and background can both be effectively improved. From the pattern recognition perspective, the optimal threshold should produce the best performance to separate the target class from the background class. This performance is characterized by intra-class variance.

Otsu's method [8] is used to automatically perform histogram shape based image thresholding. The algorithm assumes that the image to be thresholded contains two classes of pixels (e.g., foreground and background) then calculates the optimum threshold separating those two classes so that their intra-class variance is minimal. It exhaustively searches for the threshold that minimizes the intra-class variance, defined as a weighted sum of variances of the two classes:

$$\sigma^2(X_T) = W_L(E(\boldsymbol{X_L}) - E(\boldsymbol{X}))^2 + W_U(E(\boldsymbol{X_U}) - E(\boldsymbol{X}))^2 \quad (20)$$

where $E(\boldsymbol{X_L})$ and $E(\boldsymbol{X_U})$ stand for the average brightness of the two sub-images thresholded by X_T. $E(\boldsymbol{X})$ is the mean brightness of the whole image. W_L and W_U stands for the fractions to indicate the number s of two classes of pixels of the whole:

$$W_L = \frac{n_L}{n} \quad (21)$$

and

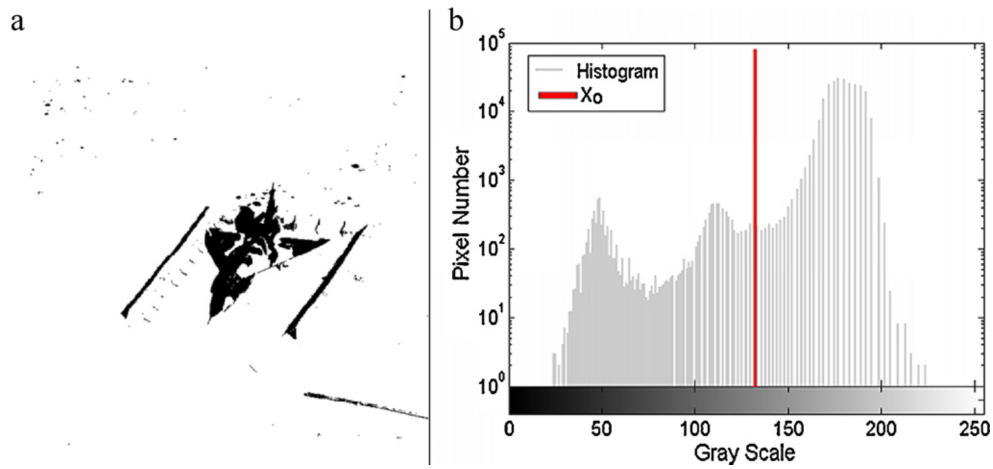

Fig. 3. Separation results using Otsu's method. (a) Separation result using X_O. (b) The location of X_O in the histogram of (a).

Thus the threshold calculated by Otsu's method can be written as

$$X_O = \arg\max_{X_T}\{\sigma^2(X_T), T = 0, 1, 2, \ldots, L-1\} \tag{22}$$

Fig. 3(a) shows the separation result of Fig. 1(a) using Otsu's method. And the location of X_O is shown in Fig. 3(b). It can be seen that Otsu's method yields a satisfactory result and the airplane is totally separated from the background.

4.2. Determinate the upper and the lower bounds for histogram equalization

The preservation of the mean brightness is of high demands in consumer electronics. Although the threshold got by Otsu's method can effectively separate the objects from the background, the mean brightness may not be strictly constrained. Additional measures must be taken to maintain the origin image brightness optimally. The mean brightness of the output image of bi-histogram equalization using X_O is as follows

$$\begin{aligned} E(\boldsymbol{Y}) &= E(\boldsymbol{Y}|\boldsymbol{X} \le X_0)p(\boldsymbol{X} \le X_0) + E(\boldsymbol{Y}|\boldsymbol{X} > X_0)p(\boldsymbol{X} > X_0) \\ &= \left(\frac{X_0 + X_O}{2}\right)\left(\sum_{i=0}^{O} p(X_i)\right) + \left(\frac{X_O + 1 + X_{L-1}}{2}\right)\left(\sum_{i=O+1}^{L-1} p(X_i)\right) \\ &= \frac{1}{2}\left[(X_0 + X_O)\left(\sum_{i=0}^{O} p(X_i)\right) + (X_O + 1 + X_{L-1})\right. \\ &\quad \left.\left(1 - \sum_{i=0}^{O} p(X_i)\right)\right] \end{aligned} \tag{23}$$

Table 1
The resulting AMBE for GHE, BBHE, DSIHE, MMBEBHE and RLBHE.

	GHE	BBHE	DSIHE	MMBEBHE	RLBHE
Aircraft	47.4781	1.4632	23.7528	0.0602	0.8624
Tank	4.8123	21.2520	5.3431	3.0595	0.7254
F16	51.8537	1.0136	18.2963	0.0236	0.7498
Plane	65.7250	16.7128	29.3686	2.8205	0.8721

The output image should keep the mean brightness of the original image as much as possible

$$E(Y) \approx E(X) = X_m = \sum_{j=0}^{L-1} X_j p(X_j) \tag{24}$$

Thus

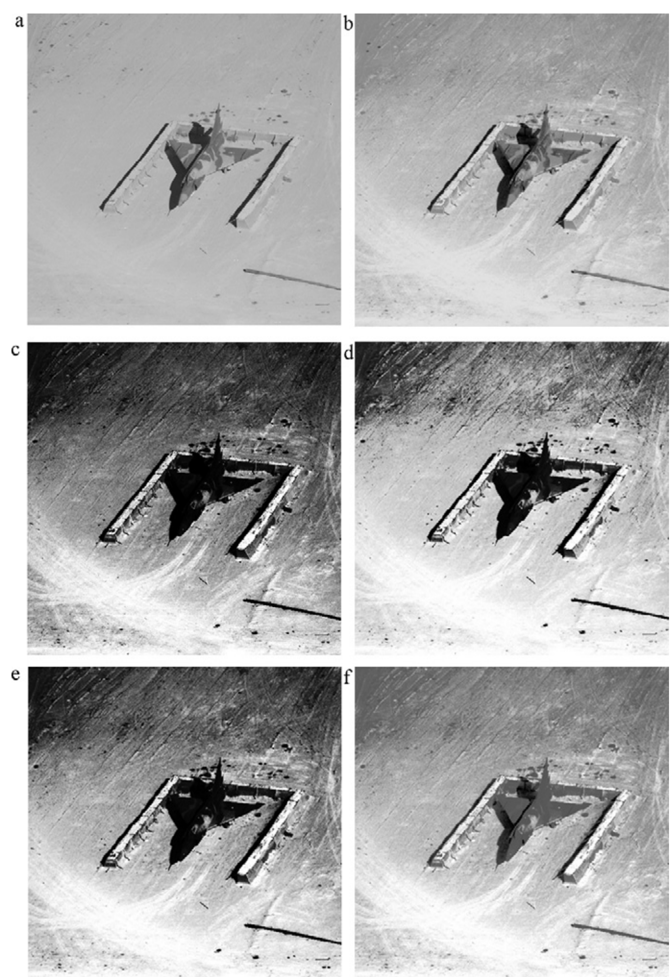

Fig. 4. (a) Original image of *Aircraft*. (b) Result of RLBHE. (c) Result of GHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE.

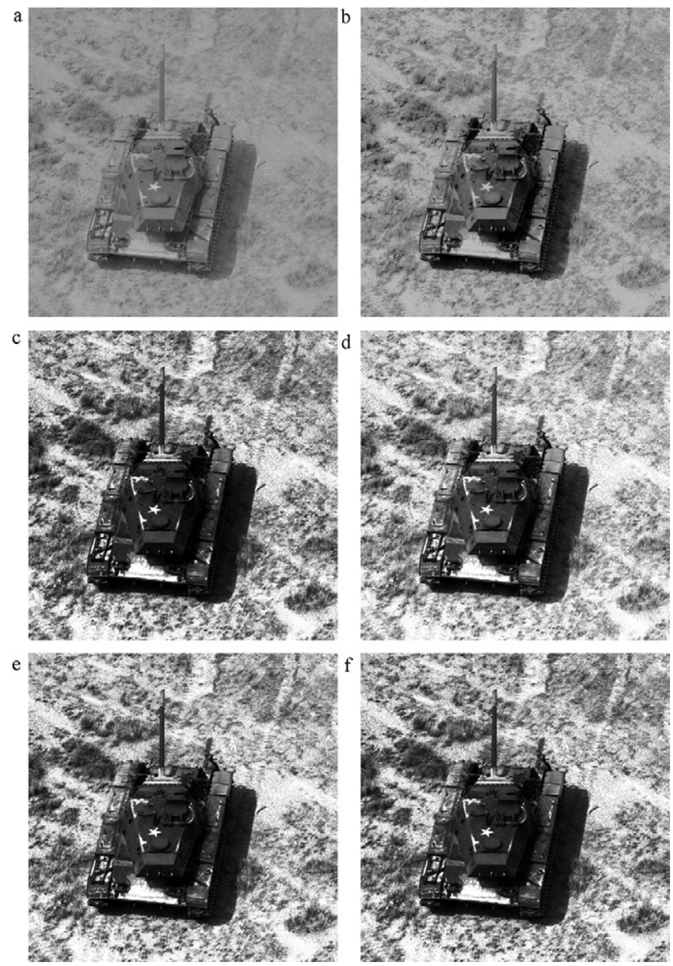

Fig. 5. (a) Original image of *Tank*. (b) Result of RLBHE. (c) Result of GHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE.

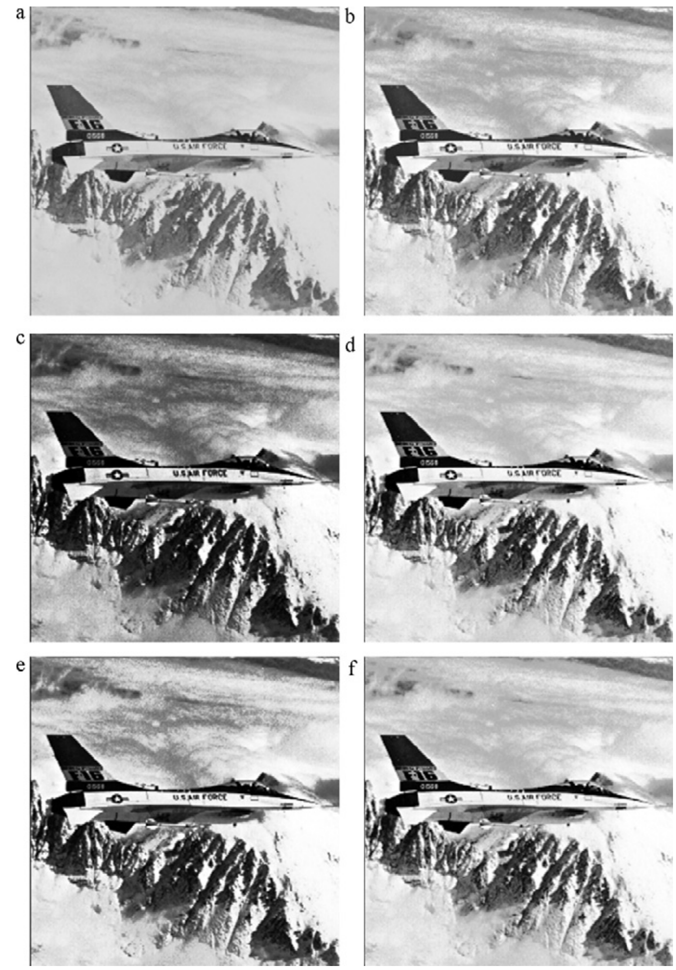

Fig. 6. (a) Original image of *F16*. (b) Result of RLBHE. (c) Result of GHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE.

$$\frac{1}{2}\left[(X_0+X_O)\left(\sum_{i=0}^{O}p(X_i)\right)+(X_O+1+X_{L-1})\left(1-\sum_{i=0}^{O}p(X_i)\right)\right]\approx X_m \tag{25}$$

From Eq. (25) we can see that, $p(X_i)$ and X_m are determined by the input image, X_O is got by Otsu's method. To make Eq. (25) holds, we can modify the range of equalized image, i.e., we replace the upper bound X_{L-1} and the lower bound X_0 with two variables X'_{L-1} and X'_0. X'_{L-1} and X'_0 are chosen to yield minimum Absolute Mean Brightness Error (AMBE) between the equalized image and the original image:

$$\begin{aligned}(X'_{L-1},X'_0) &= \underset{X'_{L-1},X'_0}{\arg\min}\{|E(\boldsymbol{Y})-E(\boldsymbol{X})|\}\\ &= \underset{X'_{L-1},X'_0}{\arg\min}\left|\frac{1}{2}\left[(X'_0+X_O)\left(\sum_{i=0}^{O}p(X_i)\right)+(X_O+1+X'_{L-1})\right.\right.\\ &\quad\left.\left.\times\left(1-\sum_{i=0}^{O}p(X_i)\right)\right]-X_m\right| = \underset{X'_{L-1},X'_0}{\arg\min}\left|X'_0\left(\sum_{i=0}^{O}p(X_i)\right)\right.\\ &\quad\left.+X'_{L-1}\left(1-\sum_{i=0}^{O}p(X_i)\right)-\left(2X_m-X_O-\left(1-\sum_{i=0}^{O}p(X_i)\right)\right)\right|\end{aligned} \tag{26}$$

where $\boldsymbol{X}$ and $\boldsymbol{Y}$ denote the input and output image, respectively, and $\sum_{i=0}^{O}p(X_i)$, X_m and X_O can be calculated beforehand, thus Eq. (26) can be simplified as

$$(X'_{L-1},X'_0)=\underset{X'_{L-1},X'_0}{\arg\min}\{(aX'_0+(1-a)X'_{L-1}-b)^2\} \tag{27}$$

where $a=\sum_{i=0}^{O}p(X_i)$, $b=2X_m-X_O-\left(1-\sum_{i=0}^{O}p(X_i)\right)$. Besides, some constraints should be applied to X'_0 and X'_{L-1}

$$\begin{cases}0\le X'_0\le X_O\\ X_O<X'_{L-1}\le X_{L-1}\end{cases} \tag{28}$$

These two constraints are obvious and we can get a optimization problem as follows.

$$(X'_{L-1},X'_0)=\underset{X'_{L-1},X'_0}{\arg\min}\{(aX'_0+(1-a)X'_{L-1}-b)^2\}\quad s.t.\begin{cases}0\le X'_0\le X_O\\ X_O<X'_{L-1}\le X_{L-1}\end{cases} \tag{29}$$

Note that this is a simple quadric optimization problem thus has a unique global optimum. The optimal X'_0 and X'_{L-1} minimize AMBE between the equalized image and the original image so that guarantee best brightness preservation.

4.3. Equalize each partition independently

The next step in RLBHE is to equalize each sub-histogram independently. This is fairly straightforward since it is same with all

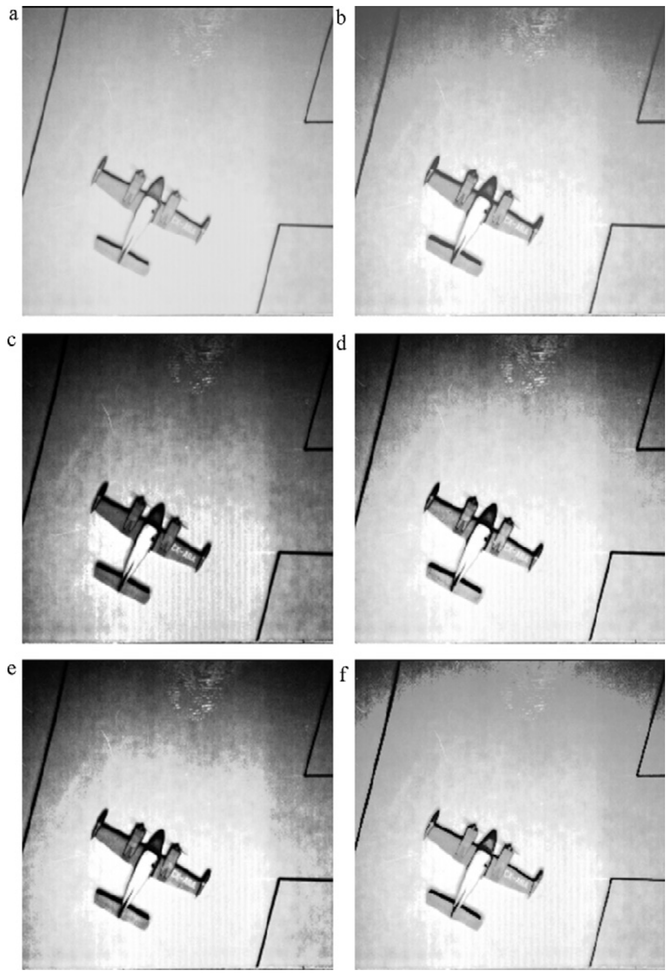

Fig. 7. (a) Original image of *Plane*. (b) Result of SWHE. (c) Result of GHE. (d) Result of BBHE. (e) Result of DSIHE. (f) Result of MMBEBHE.

bi-histogram equalization methods except for the mapping range. The final transform functions of this section are as follows:

$$f_L(X_k) = X'_0 + (X_O - X'_0)c_L(X_k), \quad k = 0, 1, \ldots, O \tag{30}$$

and

$$f_U(X_k) = X_{O+1} + (X'_{L-1} - X_{O+1})c_U(X_k), \quad k = O + 1, 1, \ldots, L - 1 \tag{31}$$

Note that the two sub-histogram are remapped to the ranges of $[X'_0, X_O]$, and $[X_{O+1}, X'_{L-1}]$. Based on the two transform functions, the decomposed sub-images are equalized independently and the composition of the resulting equalized sub-images constitute the output of RLBHE. That is, the output image of RLBHE, $\boldsymbol{Y}$, is finally expressed as

$$\boldsymbol{Y} = \{Y(i,j)\} = \boldsymbol{Y_L} \cup \boldsymbol{Y_U} = f_L(\boldsymbol{X_L}) \cup f_U(\boldsymbol{Y_U}) \tag{32}$$

where

$$\boldsymbol{Y_L} = f_L(\boldsymbol{X}) = \{f(X(i,j)|\forall X(i,j) \in \boldsymbol{X_L}\} \tag{33}$$

and

$$\boldsymbol{Y_U} = f_U(X) = \{f(X(i,j)|\forall X(i,j) \in \boldsymbol{X_U}\} \tag{34}$$

5. Results and discussion

In addition to RLBHE, we also implement GHE and three other bi-histogram equalization methods, which are BBHE, DSIHE, and MMBEBHE to demonstrate the performance of the proposed method. Table 1 lists the resulting AMBE for each of the above algorithms.

The first test image is *Aircraft* (Fig. 4), GHE, BBHE, and DSIHE tend to over enhance the take-off trail of the aircraft. MMBEBHE enhances the background's contrast well, but changes the pattern on the aircraft body. Result from RLBHE indicates that, not only the details of the trail are enhanced but also the contrast of the aircraft is significantly improved.

The second test image is *Tank* (Fig. 5). The tank has almost the same intensity with its background. Observe that resulting images of GHE, BBHE, DSIHE, and MMBEBHE have mean brightness much brighter compared to the original image and hence, results in unpleasant artifacts in the over-equalized background. Also the tank region's contrast is reduced. These artifacts are not seen with RLBHE. RLBHE has preserved the brightness very well and yielded a more natural enhancement.

The test image *F16* (Fig. 6) has been used in [3] to demonstrate the success of BBHE. Note that the output of BBHE, MMBEBHE, and RLBHE is very similar while the result of GHE and DSIHE shows obvious change in brightness (darker) and decrease of contrast around the letters "F16".

The test image *Plane* (Fig. 7) is chosen as the representative of images with high mean brightness (bright background). It is observed that the results of GHE, BBHE, and DSIHE are too dark when compared to the original image. MMBEBHE gives a more satisfactory result, but the brightness is not kept well and there are some artifacts near the borders of the image. The result of RLBHE shows that the proposed algorithm has preserved the brightness well and gives natural enhancement in most part of the image.

6. Conclusions

In this paper, we have proposed a novel contrast enhancement method using the Range Limited Bi-Histogram Equalization. RLBHE separates the histogram using Otsu's method. Unlike BBHE DSIHE and MMBEBHE, RLBHE limits the range of the equalized image to keep the input mean intensity. Therefore, the proposed method can achieve visually more pleasing contrast enhancement while maintaining the input brightness. Furthermore, similar to other histogram equalization based algorithms, RLBHE is easy to implement in real-time processing.

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